TABLE 5.4

Summary of Time Value Calculations

I. Symbols:

PV = Present value, what future cash flows are worth today

FV_t = Future value, what cash flows are worth in the future

r = Interest rate, rate of return, or discount rate per period—typically, but not always, one year

t = Number of periods—typically, but not always, the number of years

C = Cash amount

II. Future value of *C* invested at *r* percent for *t* periods:

$FV_t = C \times (1 + r)^t$

The term $(1 + r)^t$ is called the *future value factor.*

III. Present value of *C* to be received in *t* periods at *r* percent per period:

$PV = C/(1 + r)^t$

The term $1/(1 + r)^t$ is called the *present value factor.*

IV. The basic present value equation giving the relationship between present and future value is:

$PV = FV_t/(1 + r)^t$

5.4 SUMMARY AND CONCLUSIONS

This chapter has introduced you to the basic principles of present value and discounted cash flow valuation. In it, we explained a number of things about the time value of money, including:

1. For a given rate of return, the value at some point in the future of an investment made today can be determined by calculating the future value of that investment.
2. The current worth of a future cash flow or series of cash flows can be determined for a given rate of return by calculating the present value of the cash flow(s) involved.
3. The relationship between present value (PV) and future value (FV) for a given rate r and time t is given by the basic present value equation:

$$PV = FV_t/(1 + r)^t$$

As we have shown, it is possible to find any one of the four components (PV, FV_t, r, or t) given the other three.

The principles developed in this chapter will figure prominently in the chapters to come. The reason for this is that most investments, whether they involve real assets or financial assets, can be analyzed using the discounted cash flow (DCF) approach. As a result, the DCF approach is broadly applicable and widely used in practice. Before going on, therefore, you might want to do some of the problems that follow.

Chapter Review and Self-Test Problems

5.1 Calculating Future Values Assume you deposit $10,000 today in an account that pays 6 percent interest. How much will you have in five years?

5.2 **Calculating Present Values** Suppose you have just celebrated your 19th birthday. A rich uncle has set up a trust fund for you that will pay you $150,000 when you turn 30. If the relevant discount rate is 9 percent, how much is this fund worth today?

5.3 **Calculating Rates of Return** You've been offered an investment that will double your money in 10 years. What rate of return are you being offered? Check your answer using the Rule of 72.

5.4 **Calculating the Number of Periods** You've been offered an investment that will pay you 9 percent per year. If you invest $15,000, how long until you have $30,000? How long until you have $45,000?

Answers to Chapter Review and Self-Test Problems

5.1 We need to calculate the future value of $10,000 at 6 percent for five years. The future value factor is:

$$1.06^5 = 1.3382$$

The future value is thus $10,000 × 1.3382 = $13,382.26.

5.2 We need the present value of $150,000 to be paid in 11 years at 9 percent. The discount factor is:

$$1/1.09^{11} = 1/2.5804 = .3875$$

The present value is thus about $58,130.

5.3 Suppose you invest, say, $1,000. You will have $2,000 in 10 years with this investment. So, $1,000 is the amount you have today, or the present value, and $2,000 is the amount you will have in 10 years, or the future value. From the basic present value equation, we have:

$$\$2{,}000 = \$1{,}000 \times (1 + r)^{10}$$
$$2 = (1 + r)^{10}$$

From here, we need to solve for *r,* the unknown rate. As shown in the chapter, there are several different ways to do this. We will take the 10th root of 2 (by raising 2 to the power of 1/10):

$$2^{(1/10)} = 1 + r$$
$$1.0718 = 1 + r$$
$$r = 7.18\%$$

Using the Rule of 72, we have $72/t = r\%$, or $72/10 = 7.2\%$, so our answer looks good (remember that the Rule of 72 is only an approximation).

5.4 The basic equation is:

$$\$30{,}000 = \$15{,}000 \times (1 + .09)^t$$
$$2 = (1 + .09)^t$$

If we solve for *t,* we get that $t = 8.04$ years. Using the Rule of 72, we get $72/9 = 8$ years, so, once again, our answer looks good. To get $45,000, verify for yourself that you will have to wait 12.75 years.

Concepts Review and Critical Thinking Questions

1. **Present Value** The basic present value equation has four parts. What are they?
2. **Compounding** What is compounding? What is discounting?
3. **Compounding and Period** As you increase the length of time involved, what happens to future values? What happens to present values?
4. **Compounding and Interest Rates** What happens to a future value if you increase the rate r? What happens to a present value?
5. **Ethical Considerations** Take a look back at Example 5.7. Is it deceptive advertising? Is it unethical to advertise a future value like this without a disclaimer?

To answer the next five questions, refer to the GMAC security we discussed to open the chapter.

6. **Time Value of Money** Why would GMAC be willing to accept such a small amount today ($500) in exchange for a promise to repay 20 times that amount ($10,000) in the future?
7. **Call Provisions** GMAC has the right to buy back the securities anytime it wishes by paying $10,000 (this is a term of this particular deal). What impact does this feature have on the desirability of this security as an investment?
8. **Time Value of Money** Would you be willing to pay $500 today in exchange for $10,000 in 30 years? What would be the key considerations in answering yes or no? Would your answer depend on who is making the promise to repay?
9. **Investment Comparison** Suppose that when GMAC offered the security for $500, the U.S. Treasury had offered an essentially identical security. Do you think it would have had a higher or lower price? Why?
10. **Length of Investment** The GMAC security is actively bought and sold on the New York Stock Exchange. If you looked in *The Wall Street Journal* today, do you think the price would exceed the $500 original price? Why? If you looked in the year 2008, do you think the price would be higher or lower than today's price? Why?

Questions and Problems

Basic
(Questions 1–15)

1. **Simple Interest versus Compound Interest** First Tappan Bank pays 5 percent simple interest on its savings account balances, whereas First Mullineaux Bank pays 5 percent interest compounded annually. If you made a $5,000 deposit in each bank, how much more money would you earn from your First Mullineaux Bank account at the end of 10 years?
2. **Calculating Future Values** For each of the following, compute the future value:

Present Value	Years	Interest Rate	Future Value
$ 2,250	30	12%	
9,310	16	9	
76,355	3	19	
183,796	7	5	

3. **Calculating Present Values** For each of the following, compute the present value:

Basic
(*continued*)

Present Value	Years	Interest Rate	Future Value
	5	4%	$ 15,451
	8	12	51,557
	19	22	886,073
	15	20	550,164

4. Calculating Interest Rates Solve for the unknown interest rate in each of the following:

Present Value	Years	Interest Rate	Future Value
$ 265	3		$ 307
360	9		761
39,000	15		136,771
46,523	30		255,810

5. Calculating the Number of Periods Solve for the unknown number of years in each of the following:

Present Value	Years	Interest Rate	Future Value
$ 625		4%	$ 1,284
810		9	4,341
18,400		23	402,662
21,500		34	173,439

6. Calculating Interest Rates Assume the total cost of a college education will be $200,000 when your child enters college in 18 years. You presently have $27,000 to invest. What annual rate of interest must you earn on your investment to cover the cost of your child's college education?

7. Calculating the Number of Periods At 6 percent interest, how long does it take to double your money? To quadruple it?

8. Calculating Interest Rates You are offered an investment that requires you to put up $12,000 today in exchange for $40,000 15 years from now. What is the annual rate of return on this investment?

9. Calculating the Number of Periods You're trying to save to buy a new $120,000 Ferrari. You have $40,000 today that can be invested at your bank. The bank pays 5.5 percent annual interest on its accounts. How long will it be before you have enough to buy the car?

10. Calculating Present Values Imprudential, Inc., has an unfunded pension liability of $650 million that must be paid in 20 years. To assess the value of the firm's stock, financial analysts want to discount this liability back to the present. If the relevant discount rate is 8.5 percent, what is the present value of this liability?

11. Calculating Present Values You have just received notification that you have won the $1 million first prize in the Centennial Lottery. However, the prize will be awarded on your 100th birthday (assuming you're around to collect), 80 years from now. What is the present value of your windfall if the appropriate discount rate is 13 percent?

12. Calculating Future Values Your coin collection contains fifty 1952 silver dollars. If your parents purchased them for their face value when they were new,

Basic (*continued*)

how much will your collection be worth when you retire in 2067, assuming they appreciate at a 4 percent annual rate?

13. **Calculating Interest Rates and Future Values** In 1895, the first U.S. Open Golf Championship was held. The winner's prize money was $150. In 2001, the winner's check was $900,000. What was the percentage increase in the winner's check over this period? If the winner's prize increases at the same rate, what will it be in 2040?

14. **Calculating Present Values** In 2001, a mechanized toy robot from the television series *Lost in Space* sold for $750. This represented a 13.86 percent annual return. For this to be true, what must the robot have sold for new in 1965?

15. **Calculating Rates of Return** Although appealing to more refined tastes, art as a collectible has not always performed so profitably. During 1995, Christie's auctioned the William de Kooning painting *Untitled*. The highest bid of $2.2 million was rejected by the owner, who had purchased the painting at the height of the art market in 1989 for $3.52 million. Had the seller accepted the bid, what would his annual rate of return have been?

Intermediate (Questions 16–20)

16. **Calculating Rates of Return** Referring to the GMAC security we discussed at the very beginning of the chapter:
    a. Based upon the $500 price, what rate was GMAC paying to borrow money?
    b. Suppose that, on December 1, 2002, this security's price was $4,800. If an investor had purchased it for $500 at the offering and sold it on this day, what annual rate of return would she have earned?
    c. If an investor had purchased the security at market on December 1, 2002, and held it until it matured, what annual rate of return would she have earned?

17. **Calculating Present Values** Suppose you are still committed to owning a $120,000 Ferrari (see Question 9). If you believe your mutual fund can achieve an 11 percent annual rate of return and you want to buy the car in 10 years on the day you turn 30, how much must you invest today?

18. **Calculating Future Values** You have just made your first $2,000 contribution to your individual retirement account. Assuming you earn a 9 percent rate of return and make no additional contributions, what will your account be worth when you retire in 45 years? What if you wait 10 years before contributing? (Does this suggest an investment strategy?)

19. **Calculating Future Values** You are scheduled to receive $30,000 in two years. When you receive it, you will invest it for six more years at 5.5 percent per year. How much will you have in eight years?

20. **Calculating the Number of Periods** You expect to receive $10,000 at graduation in two years. You plan on investing it at 12 percent until you have $120,000. How long will you wait from now?

S&P Problems

STANDARD &POOR'S

1. **Calculating Future Values** Find the monthly adjusted prices for Tyco International LTD (TYC). If the stock appreciates 11 percent per year, what stock price do you expect to see in five years? In 10 years? Ignore dividends in your calculations.

2. Calculating Interest Rates Find the monthly adjusted prices for Redhook Ale Brewery Inc. (HOOK). What is the average annual return over the past four years?

3. Calculating the Number of Periods Find the monthly adjusted stock prices for Nucor Corp. (NUE). You find an analyst who projects the stock price will increase 12 percent per year for the foreseeable future. Based on the most recent monthly stock price, if the projection holds true, when will the stock price reach $150? When will it reach $200?

What's On the Web?

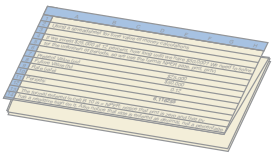
5.1 Calculating Future Values Go to www.dinkytown.net and follow the "Savings Calculator" link. If you currently have $10,000 and invest this money at 9 percent, how much will you have in 30 years? Assume you will not make any additional contributions. How much will you have if you can earn 11 percent?

5.2 Calculating the Number of Periods Go to www.dinkytown.net and follow the "Cool Million" link. You want to be a millionaire. You can earn 11.5 percent per year. Using your current age, at what age will you become a millionaire if you have $25,000 to invest, assuming you make no other deposits (ignore inflation)?

5.3 Calculating the Number of Periods Cigna has a financial calculator available at www.cigna.com. To get to the calculator, follow the "Calculator & Tools" link, then the "Present/Future Value Calculator" link. You want to buy a Lamborghini Diablo VTTT. The current market price of the car is $330,000 and you have $33,000. If you can earn an 11 percent return, how many years until you can buy this car (assuming the price stays the same)?

5.4 Calculating Rates of Return Use the Cigna financial calculator to solve the following problem. You still want to buy the Lamborghini VTTT, but you have $50,000 to deposit and want to buy the car in 15 years. What interest rate do you have to earn to accomplish this (assuming the price stays the same)?

5.5 Future Values and Taxes Taxes can greatly affect the future value of your investment. The Financial Calculators web site at www.fincalc.com has a financial calculator that adjusts your return for taxes. Follow the "Projected Savings" link on this page to find this calculator. Suppose you have $50,000 to invest today. If you can earn a 12 percent return and no additional annual savings, how much will you have in 20 years? (Enter 0 percent as the tax rate.) Now, assume that your marginal tax rate is 27.5 percent. How much will you have at this tax rate?



Spreadsheet Templates 5–1, 5–2, 5–3, 5–4, 5–5