

CONCEPT QUESTIONS

- 6.4a** What is a pure discount loan? An interest-only loan?
6.4b What does it mean to amortize a loan?
6.4c What is a balloon payment? How do you determine its value?

SUMMARY AND CONCLUSIONS**6.5**

This chapter rounds out your understanding of fundamental concepts related to the time value of money and discounted cash flow valuation. Several important topics were covered, including:

1. There are two ways of calculating present and future values when there are multiple cash flows. Both approaches are straightforward extensions of our earlier analysis of single cash flows.
2. A series of constant cash flows that arrive or are paid at the end of each period is called an ordinary annuity, and we described some useful shortcuts for determining the present and future values of annuities.
3. Interest rates can be quoted in a variety of ways. For financial decisions, it is important that any rates being compared be first converted to effective rates. The relationship between a quoted rate, such as an annual percentage rate (APR), and an effective annual rate (EAR) is given by:

$$\text{EAR} = [1 + (\text{Quoted rate}/m)]^m - 1$$

where m is the number of times during the year the money is compounded or, equivalently, the number of payments during the year.

4. Many loans are annuities. The process of providing for a loan to be paid off gradually is called amortizing the loan, and we discussed how amortization schedules are prepared and interpreted.

The principles developed in this chapter will figure prominently in the chapters to come. The reason for this is that most investments, whether they involve real assets or financial assets, can be analyzed using the discounted cash flow (DCF) approach. As a result, the DCF approach is broadly applicable and widely used in practice. For example, the next two chapters show how to value bonds and stocks using an extension of the techniques presented in this chapter. Before going on, therefore, you might want to do some of the problems that follow.

Chapter Review and Self-Test Problems

- 6.1 Present Values with Multiple Cash Flows** A first-round draft choice quarterback has been signed to a three-year, \$25 million contract. The details provide for an immediate cash bonus of \$2 million. The player is to receive \$5 million in salary at the end of the first year, \$8 million the next, and \$10 million at the end of the last year. Assuming a 15 percent discount rate, is this package worth \$25 million? How much is it worth?
- 6.2 Future Value with Multiple Cash Flows** You plan to make a series of deposits in an individual retirement account. You will deposit \$1,000 today, \$2,000 in two years, and \$2,000 in five years. If you withdraw \$1,500 in three years and



- \$1,000 in seven years, assuming no withdrawal penalties, how much will you have after eight years if the interest rate is 7 percent? What is the present value of these cash flows?
- 6.3 Annuity Present Value** You are looking into an investment that will pay you \$12,000 per year for the next 10 years. If you require a 15 percent return, what is the most you would pay for this investment?
- 6.4 APR versus EAR** The going rate on student loans is quoted as 8 percent APR. The terms of the loans call for monthly payments. What is the effective annual rate (EAR) on such a student loan?
- 6.5 It's the Principal That Matters** Suppose you borrow \$10,000. You are going to repay the loan by making equal annual payments for five years. The interest rate on the loan is 14 percent per year. Prepare an amortization schedule for the loan. How much interest will you pay over the life of the loan?
- 6.6 Just a Little Bit Each Month** You've recently finished your MBA at the Darnit School. Naturally, you must purchase a new BMW immediately. The car costs about \$21,000. The bank quotes an interest rate of 15 percent APR for a 72-month loan with a 10 percent down payment. You plan on trading the car in for a new one in two years. What will your monthly payment be? What is the effective interest rate on the loan? What will the loan balance be when you trade the car in?

Answers to Chapter Review and Self-Test Problems

- 6.1** Obviously, the package is not worth \$25 million because the payments are spread out over three years. The bonus is paid today, so it's worth \$2 million. The present values for the three subsequent salary payments are:

$$\begin{aligned}
 (\$5/1.15) + (8/1.15^2) + (10/1.15^3) &= (\$5/1.15) + (8/1.32) + (10/1.52) \\
 &= \underline{\underline{\$16.9721 \text{ million}}}
 \end{aligned}$$

The package is worth a total of \$18.9721 million.

- 6.2** We will calculate the future values for each of the cash flows separately and then add them up. Notice that we treat the withdrawals as negative cash flows:

$$\begin{aligned}
 \$1,000 \times 1.07^8 &= \$1,000 \times 1.7812 = \underline{\underline{\$ 1,718.19}} \\
 \$2,000 \times 1.07^6 &= \$2,000 \times 1.5007 = \underline{\underline{3,001.46}} \\
 -\$1,500 \times 1.07^5 &= -\$1,500 \times 1.4026 = \underline{\underline{-2,103.83}} \\
 \$2,000 \times 1.07^3 &= \$2,000 \times 1.2250 = \underline{\underline{2,450.09}} \\
 -\$1,000 \times 1.07^1 &= -\$1,000 \times 1.0700 = \underline{\underline{-1,070.00}} \\
 \text{Total future value} &= \underline{\underline{\$ 3,995.91}}
 \end{aligned}$$

This value includes a small rounding error.

To calculate the present value, we could discount each cash flow back to the present or we could discount back a single year at a time. However, because we already know that the future value in eight years is \$3,995.91, the easy way to get the PV is just to discount this amount back eight years:

$$\begin{aligned}
 \text{Present value} &= \$3,995.91/1.07^8 \\
 &= \$3,995.91/1.7182 \\
 &= \underline{\underline{\$2,325.64}}
 \end{aligned}$$

We again ignore a small rounding error. For practice, you can verify that this is what you get if you discount each cash flow back separately.

- 6.3** The most you would be willing to pay is the present value of \$12,000 per year for 10 years at a 15 percent discount rate. The cash flows here are in ordinary annuity form, so the relevant present value factor is:

$$\begin{aligned}\text{Annuity present value factor} &= (1 - \text{Present value factor})/r \\ &= [1 - (1/1.15^{10})]/.15 \\ &= (1 - .2472)/.15 \\ &= 5.0188\end{aligned}$$

The present value of the 10 cash flows is thus:

$$\begin{aligned}\text{Present value} &= \$12,000 \times 5.0188 \\ &= \$60,225\end{aligned}$$

This is the most you would pay.

- 6.4** A rate of 8 percent APR with monthly payments is actually $8\%/12 = .67\%$ per month. The EAR is thus:

$$\text{EAR} = [1 + (.08/12)]^{12} - 1 = 8.30\%$$

- 6.5** We first need to calculate the annual payment. With a present value of \$10,000, an interest rate of 14 percent, and a term of five years, the payment can be determined from:

$$\begin{aligned}\$10,000 &= \text{Payment} \times \{[1 - (1/1.14^5)]/.14\} \\ &= \text{Payment} \times 3.4331\end{aligned}$$

Therefore, the payment is $\$10,000/3.4331 = \$2,912.84$ (actually, it's \$2,912.8355; this will create some small rounding errors in the following schedule). We can now prepare the amortization schedule as follows:

Year	Beginning Balance	Total Payment	Interest Paid	Principal Paid	Ending Balance
1	\$10,000.00	\$2,912.84	\$1,400.00	\$1,512.84	\$8,487.16
2	8,487.16	2,912.84	1,188.20	1,724.63	6,762.53
3	6,762.53	2,912.84	946.75	1,966.08	4,796.45
4	4,796.45	2,912.84	671.50	2,241.33	2,555.12
5	2,555.12	2,912.84	357.72	2,555.12	0.00
Totals		\$14,564.17	\$4,564.17	\$10,000.00	

- 6.6** The cash flows on the car loan are in annuity form, so we only need to find the payment. The interest rate is $15\%/12 = 1.25\%$ per month, and there are 72 months. The first thing we need is the annuity factor for 72 periods at 1.25 percent per period:

$$\begin{aligned}\text{Annuity present value factor} &= (1 - \text{Present value factor})/r \\ &= [1 - (1/1.0125^{72})]/.0125 \\ &= [1 - (1/2.4459)]/.0125 \\ &= (1 - .4088)/.0125 \\ &= 47.2925\end{aligned}$$

The present value is the amount we finance. With a 10 percent down payment, we will be borrowing 90 percent of \$21,000, or **\$18,900**. So, to find the payment, we need to solve for C in the following:

$$\begin{aligned} \$18,900 &= C \times \text{Annuity present value factor} \\ &= C \times 47.2925 \end{aligned}$$

Rearranging things a bit, we have:

$$\begin{aligned} C &= \$18,900 \times (1/47.2925) \\ &= \$18,900 \times .02115 \\ &= \mathbf{\$399.64} \end{aligned}$$

Your payment is just under \$400 per month.

The actual interest rate on this loan is 1.25 percent per month. Based on our work in the chapter, we can calculate the effective annual rate as:

$$\text{EAR} = (1.0125)^{12} - 1 = \mathbf{16.08\%}$$

The effective rate is about one point higher than the quoted rate.

To determine the loan balance in two years, we could amortize the loan to see what the balance is at that time. This would be fairly tedious to do by hand. Using the information already determined in this problem, we can instead simply calculate the present value of the remaining payments. After two years, we have made 24 payments, so there are $72 - 24 = 48$ payments left. What is the present value of 48 monthly payments of \$399.64 at 1.25 percent per month? The relevant annuity factor is:

$$\begin{aligned} \text{Annuity present value factor} &= (1 - \text{Present value factor})/r \\ &= [1 - (1/1.0125^{48})]/.0125 \\ &= [1 - (1/1.8154)]/.0125 \\ &= (1 - .5509)/.0125 \\ &= 35.9315 \end{aligned}$$

The present value is thus:

$$\text{Present value} = \$399.64 \times 35.9315 = \$14,359.66$$

You will owe about \$14,360 on the loan in two years.

Concepts Review and Critical Thinking Questions

- Annuity Factors** There are four pieces to an annuity present value. What are they?
- Annuity Period** As you increase the length of time involved, what happens to the present value of an annuity? What happens to the future value?
- Interest Rates** What happens to the future value of an annuity if you increase the rate r ? What happens to the present value?
- Present Value** What do you think about the Tri-State Megabucks lottery discussed in the chapter advertising a \$500,000 prize when the lump-sum option is \$250,000? Is it deceptive advertising?
- Present Value** If you were an athlete negotiating a contract, would you want a big signing bonus payable immediately and smaller payments in the future, or vice versa? How about looking at it from the team's perspective?

6. **Present Value** Suppose two athletes sign 10-year contracts for \$80 million. In one case, we're told that the \$80 million will be paid in 10 equal installments. In the other case, we're told that the \$80 million will be paid in 10 installments, but the installments will increase by 5 percent per year. Who got the better deal?
7. **APR and EAR** Should lending laws be changed to require lenders to report EARs instead of APRs? Why or why not?
8. **Time Value** On subsidized Stafford loans, a common source of financial aid for college students, interest does not begin to accrue until repayment begins. Who receives a bigger subsidy, a freshman or a senior? Explain.
9. **Time Value** In words, how would you go about valuing the subsidy on a subsidized Stafford loan?
10. **Time Value** Eligibility for a subsidized Stafford loan is based on current financial need. However, both subsidized and unsubsidized Stafford loans are repaid out of future income. Given this, do you see a possible objection to having two types?

Questions and Problems

1. **Present Value and Multiple Cash Flows** Mercer Shaved Ice Co. has identified an investment project with the following cash flows. If the discount rate is 10 percent, what is the present value of these cash flows? What is the present value at 18 percent? At 24 percent?

Basic
(Questions 1–28)

Year	Cash Flow
1	\$1,300
2	500
3	700
4	1,620

2. **Present Value and Multiple Cash Flows** Investment X offers to pay you \$3,000 per year for eight years, whereas Investment Y offers to pay you \$5,000 per year for four years. Which of these cash flow streams has the higher present value if the discount rate is 5 percent? If the discount rate is 22 percent?
3. **Future Value and Multiple Cash Flows** Rasputin, Inc., has identified an investment project with the following cash flows. If the discount rate is 8 percent, what is the future value of these cash flows in Year 4? What is the future value at a discount rate of 11 percent? At 24 percent?

Year	Cash Flow
1	\$ 900
2	1,000
3	1,100
4	1,200

4. **Calculating Annuity Present Value** An investment offers \$4,100 per year for 15 years, with the first payment occurring one year from now. If the required return is 10 percent, what is the value of the investment? What would the value be if the payments occurred for 40 years? For 75 years? Forever?
5. **Calculating Annuity Cash Flows** If you put up \$20,000 today in exchange for a 8.25 percent, 12-year annuity, what will the annual cash flow be?

Basic*(continued)*

6. **Calculating Annuity Values** Your company will generate \$75,000 in annual revenue each year for the next eight years from a new information database. The computer system needed to set up the database costs \$380,000. If you can borrow the money to buy the computer system at 7.5 percent annual interest, can you afford the new system?
7. **Calculating Annuity Values** If you deposit \$1,500 at the end of each of the next 20 years into an account paying 9.5 percent interest, how much money will you have in the account in 20 years? How much will you have if you make deposits for 40 years?
8. **Calculating Annuity Values** You want to have \$50,000 in your savings account five years from now, and you're prepared to make equal annual deposits into the account at the end of each year. If the account pays 6.2 percent interest, what amount must you deposit each year?
9. **Calculating Annuity Values** Biktimirov Bank offers you a \$35,000, seven-year term loan at 10 percent annual interest. What will your annual loan payment be?
10. **Calculating Perpetuity Values** The Perpetual Life Insurance Co. is trying to sell you an investment policy that will pay you and your heirs \$5,000 per year forever. If the required return on this investment is 9 percent, how much will you pay for the policy?
11. **Calculating Perpetuity Values** In the previous problem, suppose the Perpetual Life Insurance Co. told you the policy costs \$58,000. At what interest rate would this be a fair deal?
12. **Calculating EAR** Find the EAR in each of the following cases:

Stated Rate (APR)	Number of Times Compounded	Effective Rate (EAR)
12%	Quarterly	
8	Monthly	
7	Daily	
16	Infinite	

13. **Calculating APR** Find the APR, or stated rate, in each of the following cases:

Stated Rate (APR)	Number of Times Compounded	Effective Rate (EAR)
	Semiannually	7.2%
	Monthly	9.1
	Weekly	18.5
	Infinite	28.3

14. **Calculating EAR** First National Bank charges 9.1 percent compounded monthly on its business loans. First United Bank charges 9.2 percent compounded semiannually. As a potential borrower, which bank would you go to for a new loan?
15. **Calculating APR** Cannone Credit Corp. wants to earn an effective annual return on its consumer loans of 14 percent per year. The bank uses daily compounding on its loans. What interest rate is the bank required by law to report to potential borrowers? Explain why this rate is misleading to an uninformed borrower.
16. **Calculating Future Values** What is the future value of \$600 in 20 years assuming an interest rate of 11 percent compounded semiannually?

- 17. Calculating Future Values** Corn Credit Bank is offering 6.3 percent compounded daily on its savings accounts. If you deposit \$5,000 today, how much will you have in the account in 5 years? In 10 years? In 20 years? **Basic** (continued)
- 18. Calculating Present Values** An investment will pay you \$19,000 in six years. If the appropriate discount rate is 12 percent compounded daily, what is the present value?
- 19. EAR versus APR** Big Al's Pawn Shop charges an interest rate of 25 percent per month on loans to its customers. Like all lenders, Big Al must report an APR to consumers. What rate should the shop report? What is the effective annual rate?
- 20. Calculating Loan Payments** You want to buy a new sports coupe for \$48,250, and the finance office at the dealership has quoted you a 9.8 percent APR loan for 60 months to buy the car. What will your monthly payments be? What is the effective annual rate on this loan?
- 21. Calculating Number of Periods** One of your customers is delinquent on his accounts payable balance. You've mutually agreed to a repayment schedule of \$400 per month. You will charge 1.5 percent per month interest on the overdue balance. If the current balance is \$17,805.69, how long will it take for the account to be paid off?
- 22. Calculating EAR** Friendly's Quick Loans, Inc., offers you "three for four or I knock on your door." This means you get \$3 today and repay \$4 when you get your paycheck in one week (or else). What's the effective annual return Friendly's earns on this lending business? If you were brave enough to ask, what APR would Friendly's say you were paying?
- 23. Valuing Perpetuities** Maybepay Life Insurance Co. is selling a perpetuity contract that pays \$1,050 monthly. The contract currently sells for \$75,000. What is the monthly return on this investment vehicle? What is the APR? The effective annual return?
- 24. Calculating Annuity Future Values** You are to make monthly deposits of \$100 into a retirement account that pays 11 percent interest compounded monthly. If your first deposit will be made one month from now, how large will your retirement account be in 20 years?
- 25. Calculating Annuity Future Values** In the previous problem, suppose you make \$1,200 annual deposits into the same retirement account. How large will your account balance be in 20 years?
- 26. Calculating Annuity Present Values** Beginning three months from now, you want to be able to withdraw \$1,000 each quarter from your bank account to cover college expenses over the next four years. If the account pays 0.75 percent interest per quarter, how much do you need to have in your bank account today to meet your expense needs over the next four years?
- 27. Discounted Cash Flow Analysis** If the appropriate discount rate for the following cash flows is 14 percent compounded quarterly, what is the present value of the cash flows?

Year	Cash Flow
1	\$ 800
2	700
3	0
4	1,200

Basic*(continued)*

28. **Discounted Cash Flow Analysis** If the appropriate discount rate for the following cash flows is 11.5 percent per year, what is the present value of the cash flows?

Year	Cash Flow
1	\$1,500
2	0
3	7,200
4	900

Intermediate*(Questions 29–59)*

29. **Simple Interest versus Compound Interest** First Simple Bank pays 6 percent simple interest on its investment accounts. If First Complex Bank pays interest on its accounts compounded annually, what rate should the bank set if it wants to match First Simple Bank over an investment horizon of 10 years?
30. **Calculating EAR** You are looking at an investment that has an effective annual rate of 14 percent. What is the effective semiannual return? The effective quarterly return? The effective monthly return?
31. **Calculating Interest Expense** You receive a credit card application from Shady Banks Savings and Loan offering an introductory rate of 2.90 percent per year, compounded monthly for the first six months, increasing thereafter to 15 percent compounded monthly. Assuming you transfer the \$3,000 balance from your existing credit card and make no subsequent payments, how much interest will you owe at the end of the first year?
32. **Calculating the Number of Periods** You are saving to buy a \$150,000 house. There are two competing banks in your area, both offering certificates of deposit yielding 5 percent. How long will it take your initial \$95,000 investment to reach the desired level at First Bank, which pays simple interest? How long at Second Bank, which compounds interest monthly?
33. **Calculating Future Values** You have an investment that will pay you 1.72 percent per month. How much will you have per dollar invested in one year? In two years?
34. **Calculating the Number of Periods** You have \$1,100 today. You need \$2,000. If you earn 1 percent per month, how many months will you wait?
35. **Calculating Rates of Return** Suppose an investment offers to quadruple your money in 12 months (don't believe it). What rate of return per quarter are you being offered?
36. **Comparing Cash Flow Streams** You've just joined the investment banking firm of Dewey, Cheatum, and Howe. They've offered you two different salary arrangements. You can have \$75,000 per year for the next two years, or you can have \$55,000 per year for the next two years, along with a \$30,000 signing bonus today. If the interest rate is 10 percent compounded monthly, which do you prefer?
37. **Calculating Present Value of Annuities** Peter Piper wants to sell you an investment contract that pays equal \$10,000 amounts at the end of each of the next 20 years. If you require an effective annual return of 9.5 percent on this investment, how much will you pay for the contract today?
38. **Calculating Rates of Return** You're trying to choose between two different investments, both of which have up-front costs of \$30,000. Investment G returns \$55,000 in six years. Investment H returns \$90,000 in 11 years. Which of these investments has the higher return?

- 39. Present Value and Interest Rates** What is the relationship between the value of an annuity and the level of interest rates? Suppose you just bought a 10-year annuity of \$2,000 per year at the current interest rate of 10 percent per year. What happens to the value of your investment if interest rates suddenly drop to 5 percent? What if interest rates suddenly rise to 15 percent?
- 40. Calculating the Number of Payments** You're prepared to make monthly payments of \$95, beginning at the end of this month, into an account that pays 10 percent interest compounded monthly. How many payments will you have made when your account balance reaches \$18,000?
- 41. Calculating Annuity Present Values** You want to borrow \$40,000 from your local bank to buy a new sailboat. You can afford to make monthly payments of \$825, but no more. Assuming monthly compounding, what is the highest rate you can afford on a 60-month APR loan?
- 42. Calculating Loan Payments** You need a 30-year, fixed-rate mortgage to buy a new home for \$180,000. Your mortgage bank will lend you the money at a 7.5 percent APR for this 360-month loan. However, you can only afford monthly payments of \$1,000, so you offer to pay off any remaining loan balance at the end of the loan in the form of a single balloon payment. How large will this balloon payment have to be for you to keep your monthly payments at \$1,000?
- 43. Calculating Present Values** In the 1994 NBA draft, no one was surprised when the Milwaukee Bucks took Glenn "Big Dog" Robinson with the first pick, but Robinson wanted big bucks from the Bucks: a 13-year deal worth a total of \$100 million. He had to settle for about \$68 million over 10 years. His contract called for \$2.9 million the first year, with annual raises of \$870,000. So, how big a bite did Big Dog really take? Assume a 10 percent discount rate.
- 44. Calculating Present Values** In our previous question, we looked at the numbers for Big Dog's basketball contract. Now let's take a look at the terms for Shaquille "Shaq" O'Neal, the number one pick in 1992 who was drafted by the Orlando Magic. Shaquille signed a seven-year contract with estimated total payments of about \$40 million. Although the precise terms were not disclosed, it was reported that Shaq would receive a salary of \$3 million the first year, with raises of \$900,000 each year thereafter. If the cash flows are discounted at the same 10 percent discount rate we used for Robinson, does the "Shaq Attack" result in the same kind of numbers? Did Robinson achieve his goal of being paid more than any other rookie in NBA history, including Shaq? Are the different contract lengths a factor? (Hint: yes.)
- 45. EAR versus APR** You have just purchased a new warehouse. To finance the purchase, you've arranged for a 30-year mortgage loan for 80 percent of the \$1,200,000 purchase price. The monthly payment on this loan will be \$9,300. What is the APR on this loan? The EAR?
- 46. Present Value and Break-Even Interest** Consider a firm with a contract to sell an asset for \$95,000 three years from now. The asset costs \$57,000 to produce today. Given a relevant discount rate on this asset of 14 percent per year, will the firm make a profit on this asset? At what rate does the firm just break even?
- 47. Present Value and Interest Rates** You've just won the U.S. Lottery. Lottery officials offer you the choice of two alternative payouts: either \$2 million today, or \$4 million 10 years from now. Which payout will you choose if the relevant discount rate is 0 percent? If it is 10 percent? If it is 20 percent?

Intermediate
(continued)

Intermediate
(continued)

48. **Calculating Present Value of Annuities** Congratulations! You've just won the \$15 million first prize in the Subscriptions R Us Sweepstakes. Unfortunately, the sweepstakes will actually give you the \$15 million in \$375,000 annual installments over the next 40 years, beginning next year. If your appropriate discount rate is 11 percent per year, how much money did you really win?
49. **Present Value and Multiple Cash Flows** What is the present value of \$1,000 per year, at a discount rate of 12 percent, if the first payment is received 8 years from now and the last payment is received 20 years from now?
50. **Variable Interest Rates** A 10-year annuity pays \$1,500 per month, and payments are made at the end of each month. If the interest rate is 15 percent compounded monthly for the first four years, and 12 percent compounded monthly thereafter, what is the present value of the annuity?
51. **Comparing Cash Flow Streams** You have your choice of two investment accounts. Investment A is a 10-year annuity that features end-of-month \$1,000 payments and has an interest rate of 11.5 percent compounded monthly. Investment B is an 8 percent continuously compounded lump-sum investment, also good for 10 years. How much money would you need to invest in B today for it to be worth as much as Investment A 10 years from now?
52. **Calculating Present Value of a Perpetuity** Given an interest rate of 6.5 percent per year, what is the value at date $t = 7$ of a perpetual stream of \$500 payments that begin at date $t = 13$?
53. **Calculating EAR** A local finance company quotes a 13 percent interest rate on one-year loans. So, if you borrow \$20,000, the interest for the year will be \$2,600. Because you must repay a total of \$22,600 in one year, the finance company requires you to pay $\$22,600/12$, or \$1,883.33, per month over the next 12 months. Is this a 13 percent loan? What rate would legally have to be quoted? What is the effective annual rate?
54. **Calculating Future Values** If today is Year 0, what is the future value of the following cash flows five years from now? What is the future value 10 years from now? Assume a discount rate of 9 percent per year.

Year	Cash Flow
2	\$30,000
3	50,000
5	85,000

55. **Calculating Present Values** A 5-year annuity of ten \$8,000 semiannual payments will begin 9 years from now, with the first payment coming 9.5 years from now. If the discount rate is 14 percent compounded monthly, what is the value of this annuity five years from now? What is the value three years from now? What is the current value of the annuity?
56. **Calculating Annuities Due** As discussed in the text, an ordinary annuity assumes equal payments at the end of each period over the life of the annuity. An *annuity due* is the same thing except the payments occur at the beginning of each period instead. Thus, a three-year annual annuity due would have periodic payment cash flows occurring at Years 0, 1, and 2, whereas a three-year annual ordinary annuity would have periodic payment cash flows occurring at Years 1, 2, and 3.

- a. At a 10.5 percent annual discount rate, find the present value of a six-year ordinary annuity contract of \$475 payments.
 b. Find the present value of the same contract if it is an annuity due.

Intermediate*(continued)*

57. **Calculating Annuities Due** You want to buy a new sports car from Muscle Motors for \$48,000. The contract is in the form of a 48-month annuity due at a 9.25 percent APR. What will your monthly payment be?
58. **Amortization with Equal Payments** Prepare an amortization schedule for a five-year loan of \$20,000. The interest rate is 12 percent per year, and the loan calls for equal annual payments. How much interest is paid in the third year? How much total interest is paid over the life of the loan?
59. **Amortization with Equal Principal Payments** Rework Problem 58 assuming that the loan agreement calls for a principal reduction of \$4,000 every year instead of equal annual payments.
60. **Discount Interest Loans** This question illustrates what is known as *discount interest*. Imagine you are discussing a loan with a somewhat unscrupulous lender. You want to borrow \$20,000 for one year. The interest rate is 11 percent. You and the lender agree that the interest on the loan will be $.11 \times \$20,000 = \$2,200$. So the lender deducts this interest amount from the loan up front and gives you \$17,800. In this case, we say that the discount is \$2,200. What's wrong here?
61. **Calculating EAR with Discount Interest** You are considering a one-year loan of \$13,000. The interest rate is quoted on a discount basis (see the previous problem) as 16 percent. What is the effective annual rate?
62. **Calculating EAR with Points** You are looking at a one-year loan of \$10,000. The interest rate is quoted as 12 percent plus three points. A *point* on a loan is simply 1 percent (one percentage point) of the loan amount. Quotes similar to this one are very common with home mortgages. The interest rate quotation in this example requires the borrower to pay three points to the lender up front and repay the loan later with 12 percent interest. What rate would you actually be paying here?
63. **Calculating EAR with Points** The interest rate on a one-year loan is quoted as 14 percent plus two points (see the previous problem). What is the EAR? Is your answer affected by the loan amount?
64. **EAR versus APR** There are two banks in the area that offer 30-year, \$150,000 mortgages at 8.5 percent and charge a \$1,000 loan application fee. However, the application fee charged by Insecurity Bank and Trust is refundable if the loan application is denied, whereas that charged by I. M. Greedy and Sons Mortgage Bank is not. The current disclosure law requires that any fees that will be refunded if the applicant is rejected be included in calculating the APR, but this is not required with nonrefundable fees (presumably because refundable fees are part of the loan rather than a fee). What are the EARs on these two loans? What are the APRs?
65. **Calculating EAR with Add-On Interest** This problem illustrates a deceptive way of quoting interest rates called *add-on interest*. Imagine that you see an advertisement for Crazy Judy's Stereo City that reads something like this: "\$1,000 Instant Credit! 14% Simple Interest! Three Years to Pay! Low, Low Monthly Payments!" You're not exactly sure what all this means and somebody has spilled ink over the APR on the loan contract, so you ask the manager for clarification.

Challenge

(Questions 60–75)

Judy explains that if you borrow \$1,000 for three years at 14 percent interest, in three years you will owe:

Challenge*(continued)*

$$\$1,000 \times 1.14^3 = \$1,000 \times 1.48154 = \$1,481.54.$$

Now, Judy recognizes that coming up with \$1,481.54 all at once might be a strain, so she lets you make “low, low monthly payments” of $\$1,481.54/36 = \41.15 per month, even though this is extra bookkeeping work for her.

Is this a 14 percent loan? Why or why not? What is the APR on this loan? What is the EAR? Why do you think this is called add-on interest?

- 66. Calculating Annuity Payments** This is a classic retirement problem. A time line will help in solving it. Your friend is celebrating her 35th birthday today and wants to start saving for her anticipated retirement at age 65. She wants to be able to withdraw \$80,000 from her savings account on each birthday for 15 years following her retirement; the first withdrawal will be on her 66th birthday. Your friend intends to invest her money in the local credit union, which offers 9 percent interest per year. She wants to make equal annual payments on each birthday into the account established at the credit union for her retirement fund.
- If she starts making these deposits on her 36th birthday and continues to make deposits until she is 65 (the last deposit will be on her 65th birthday), what amount must she deposit annually to be able to make the desired withdrawals at retirement?
 - Suppose your friend has just inherited a large sum of money. Rather than making equal annual payments, she has decided to make one lump-sum payment on her 35th birthday to cover her retirement needs. What amount does she have to deposit?
 - Suppose your friend’s employer will contribute \$1,500 to the account every year as part of the company’s profit-sharing plan. In addition, your friend expects a \$30,000 distribution from a family trust fund on her 55th birthday, which she will also put into the retirement account. What amount must she deposit annually now to be able to make the desired withdrawals at retirement?
- 67. Calculating the Number of Periods** Your Christmas ski vacation was great, but it unfortunately ran a bit over budget. All is not lost, because you just received an offer in the mail to transfer your \$10,000 balance from your current credit card, which charges an annual rate of 17.9 percent, to a new credit card charging a rate of 8.9 percent. How much faster could you pay the loan off by making your planned monthly payments of \$200 with the new card? What if there was a 2 percent fee charged on any balances transferred?
- 68. Future Value and Multiple Cash Flows** An insurance company is offering a new policy to its customers. Typically, the policy is bought by a parent or grandparent for a child at the child’s birth. The details of the policy are as follows: The purchaser (say, the parent) makes the following six payments to the insurance company:
- | | |
|------------------|-------|
| First birthday: | \$750 |
| Second birthday: | \$750 |
| Third birthday: | \$850 |
| Fourth birthday: | \$850 |
| Fifth birthday: | \$950 |
| Sixth birthday: | \$950 |

After the child's sixth birthday, no more payments are made. When the child reaches age 65, he or she receives \$175,000. If the relevant interest rate is 10 percent for the first six years and 6 percent for all subsequent years, is the policy worth buying?

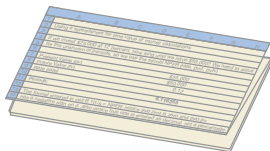
Challenge
(continued)

- 69. Calculating a Balloon Payment** You have just arranged for a \$300,000 mortgage to finance the purchase of a large tract of land. The mortgage has a 9 percent APR, and it calls for monthly payments over the next 15 years. However, the loan has a five-year balloon payment, meaning that the loan must be paid off then. How big will the balloon payment be?
- 70. Calculating Interest Rates** A financial planning service offers a college savings program. The plan calls for you to make six annual payments of \$5,000 each, with the first payment occurring today, your child's 12th birthday. Beginning on your child's 18th birthday, the plan will provide \$15,000 per year for four years. What return is this investment offering?
- 71. Break-Even Investment Returns** Your financial planner offers you two different investment plans. Plan X is an \$8,000 annual perpetuity. Plan Y is a 10-year, \$20,000 annual annuity. Both plans will make their first payment one year from today. At what discount rate would you be indifferent between these two plans?
- 72. Perpetual Cash Flows** What is the value of an investment that pays \$5,200 every *other* year forever, if the first payment occurs one year from today and the discount rate is 14 percent compounded daily? What is the value today if the first payment occurs four years from today?
- 73. Ordinary Annuities and Annuities Due** As discussed in the text, an annuity due is identical to an ordinary annuity except that the periodic payments occur at the beginning of each period and not at the end of the period (see Question 56). Show that the relationship between the value of an ordinary annuity and the value of an otherwise equivalent annuity due is:
- $$\text{Annuity due value} = \text{Ordinary annuity value} \times (1 + r)$$
- Show this for both present and future values.
- 74. Calculating Annuities Due** A 10-year annual annuity due with the first payment occurring at date $t = 7$ has a current value of \$50,000. If the discount rate is 13 percent per year, what is the annuity payment amount?
- 75. Calculating EAR** A check-cashing store is in the business of making personal loans to walk-up customers. The store makes only one-week loans at 11 percent interest per week.
- What APR must the store report to its customers? What is the EAR that the customers are actually paying?
 - Now suppose the store makes one-week loans at 11 percent discount interest per week (see Question 60). What's the APR now? The EAR?
 - The check-cashing store also makes one-month add-on interest loans at 8 percent discount interest per week. Thus, if you borrow \$100 for one month (four weeks), the interest will be $(\$100 \times 1.08^4) - 100 = \36.05 . Because this is discount interest, your net loan proceeds today will be \$63.95. You must then repay the store \$100 at the end of the month. To help you out, though, the store lets you pay off this \$100 in installments of \$25 per week. What is the APR of this loan? What is the EAR?



What's On the Web?

- 6.1 Annuity Future Value** The St. Louis Federal Reserve Board has files listing historical interest rates on their web site www.stls.frb.org. Follow the link for “FRED”/data, then “Interest Rates.” You will find listings for Moody’s Seasoned Aaa Corporate Bond Yield and Moody’s Seasoned Baa Corporate Bond Yield. (These rates are discussed in the next chapter.) If you invest \$2,000 per year for the next 40 years at the most recent Aaa yield, how much will you have? What if you invest the same amount at the Baa yield?
- 6.2 Loan Payments** Finding the time necessary until you pay off a loan is simple if you make equal payments each month. However, when paying off credit cards many individuals only make the minimum monthly payment, which is generally \$10 or 2 percent to 3 percent of the balance, whichever is greater. You can find a credit card calculator at www.fincalc.com. You currently owe \$10,000 on a credit card with a 17 percent interest rate and a minimum payment of \$10 or 2 percent of your balance, whichever is greater. How soon will you pay off this debt if you make the minimum payment each month? How much total interest will you pay?
- 6.3 Annuity Payments** Go to www.fcfcorp.com/onlinecalc.htm. Use the calculator to solve this problem. If you have \$1,500,000 when you retire and want to withdraw an equal amount for the next 30 years, how much can you withdraw each year if you earn 7 percent? What if you earn 9 percent?
- 6.4 Annuity Payments** The St. Louis Federal Reserve Board has files listing historical interest rates on their web site www.stls.frb.org. Follow the link for “FRED”/data, then “Interest Rates.” You will find a listing for the Bank Prime Loan Rate. The file lists the monthly prime rate since January 1949 (1949.01). What is the most recent prime rate? What is the highest prime rate over this period? If you bought a house for \$150,000 at the current prime rate on a 30-year mortgage with monthly payments, how much are your payments? If you had purchased the house at the same price when the prime rate was its highest, what would your monthly payments have been?
- 6.5 Loan Amortization** CMB Mortgage Services, located at www.cmbmortgage.com, has a financial calculator that will prepare an amortization table based on your inputs. Follow the “Mortgage Calculator” link and then “What are my monthly payments?” link. You want to buy a home for \$200,000 on a 30-year mortgage with monthly payments at the rate quoted on the site. What percentage of your first month’s payment is principal? What percentage of your last month’s payment is principal? What is the total interest paid on the loan?



Spreadsheet Templates 6–1, 6–3, 6–6, 6–10, 6–14, 6–15, 6–17, 6–36, 6–42, 6–45, 6–46, 6–50, 6–66, 6–70, 6–71